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# Half quantized Hall effect

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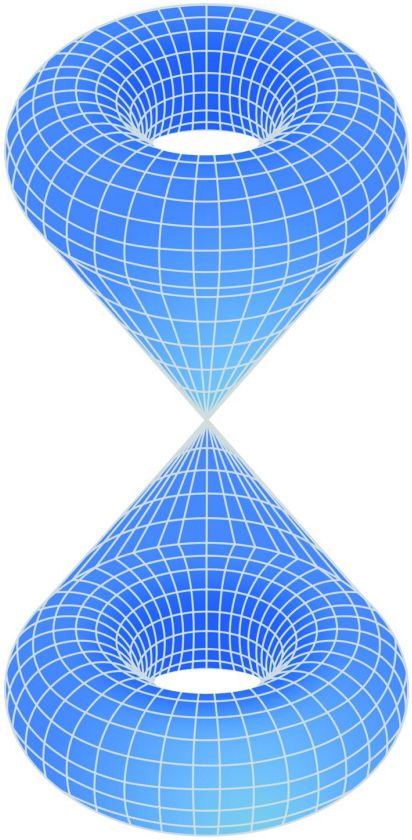
## Author introduction

Shun-Qing Shen is a professor in the Department of Physics at the University of Hong Kong. He is an expert in the field of condensed matter physics and is distinguished for his research works on topological insulators, quantum transport, and novel quantum states of condensed matter. He published a single-authored monograph, *Topological Insulators* (Springer, 1st ed., 2012; 2nd ed., 2017). He received his BS, MS, and PhD in theoretical physics from Fudan University in Shanghai, China. He was a post-doctoral fellow at China Center of Advanced Science and Technology (CCAST), Beijing, China, an Alexander von Humboldt fellow at Max Planck Institute for Physics of Complex Systems, Dresden, Germany, and a JSPS research fellow at Tokyo Institute of Technology, Japan.



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# Half quantized Hall effect

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**Abstract** Here we report a half-quantized Hall effect in a metal or semimetal. The Hall conductance is half quantized and the longitudinal conductance is nonzero. Consequently, the Hall resistivity is not quantized. The half quantization occurs when the parity symmetry or time reversal symmetry emerges near the Fermi surface or Fermi level while the symmetry is broken in the whole system. A recent experiment reports the observation of the half-quantized Hall conductance in a magnetically-doped topological insulator. We discover that a single gapless Dirac cone exists in the band structure and has half-quantized conductance when the Fermi level intercepts the gapless surface states in which the parity symmetry is invariant. As there are no localized chiral edge states in the gapless and metallic system, we find that the chiral edge current is carried by the gapless surface states. The current density peaks at the edge and decays in a power law rather than the exponential decay as in the conventional quantum anomalous Hall effect. The half quantized Hall conductance is a signature of parity anomaly in a single gapless Dirac cone on a lattice. We term the nontrivial quantum phase as “parity anomalous semimetal”. The work opens the door to exploring novel topological states of matter with fractional topological invariants.

**Keywords** Half quantized Hall effect, parity anomaly, gapless Dirac cone, half-quantized Hall conductance, topological metallic phase

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